On signals compression by EMD

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A new signals coding framework based on empirical mode decomposition (EMD) is introduced. EMD breaks down any signal into a reduced number of oscillating components called intrinsic modes functions (IMFs). Based on IMF properties, different coding strategies are presented. No assumptions concerning the linearity or the stationarity are made about the signal to be coded. Results obtained on ECG signals are presented and compared to those of wavelet coding.

Introduction: Most of the compression strategies use signal expansions such as subband coding or transform coding [1]. In subband coding the signal is decomposed into a set of band-limited (subbands) components. Even good bit rates are obtained, and this class of coding uses pre-determined basis functions. Unfortunately, using fixed basis functions prevents the decomposition from being parsimonious for any kind of signals. Actually, even if a basis is well suited for a class of signals, in the sense that it yields compact descriptions with a few significant terms, there are other signals for which the basis under consideration performs poorly. Thus, there is a need for data driven coding. Recently, a new expansion, referred to as empirical mode decomposition (EMD) has been introduced for analysing signals in a totally adaptive way [2]. This decomposition relies on no *a priori* choice of basis functions and the extracted oscillating modes, called intrinsic mode functions (IMFs), are fully described by their local extrema [2]. The EMD breaks down a signal, x(t), into K modes, as follows [2]:

$$\mathrm{x}(t) = \sum_{k=1}^{K} \mathrm{IMF}_k(t) + \mathrm{r}_K(t) = \sum_{k=1}^{K} \mathrm{a}_k(t)e^{i\phi_k(t)} + \mathrm{r}_K(t) \qquad (1)$$

where $\mathrm{r}_K(t)$ is the residual, $\mathrm{a}_k(t)$ are the instantaneous amplitude (IA) and the instantaneous phase (IP) of the kth IMF. We have shown that extrema of IMFs can be used for audio coding purposes [3]. In this Letter we show that this approach can be extended to encode any signal and propose different coding strategies based on IMF properties.

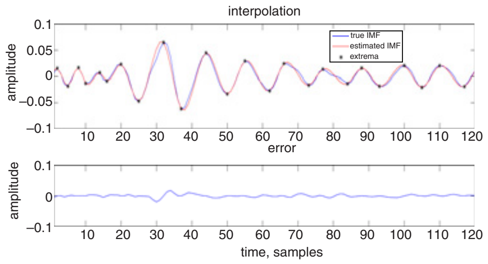

Fig. 1 *Original IMF and estimated version by spline interpolation of extrema*

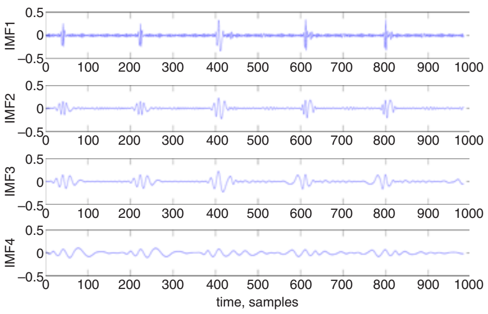

Fig. 2 *Decomposition of ECG signal by EMD (4 out of 8 IMFs)*

Extrema coding: An IMF can be represented by its extrema. Fig. 1 shows an example of an IMF and its approximate version by interpolation of its extrema with negligible reconstruction error. This example illustrates the interest of encoding extrema. The number of extrema of each IMF can be reduced using an appropriate thresholding. Only extrema saved in the coding are those that extend above a threshold. Fig. 2 shows an example of an EGC signal decomposed by EMD. Extrema of IMFs are given in Table 1 where, by reconstruction, the number of extrema decreases from one IMF to the next one. The way this number varies from IMF to IMF is exploited in the unequal bit allocation strategy. The extrema amplitude of each IMF is scaled by a factor equal to the maximum of the extrema amplitude value. We quantise the position extrema, scaling factor and extrema amplitude value by a scalar quantisation. Both minima and maxima are encoded.

Table 1: Number of extrema per IMF of ECG signal

IMF	1	2	3	4	5	6	7	8
Extrema	140	45	20	14	9	6	4	2

Envelope coding: One step of EMD is to identify local maxima (minima) and to connect them by a spline to form an upper (lower) envelope. The aim is to remove the asymmetry between these envelopes in order to transform the original signal into an AM signal (Fig. 3). Compared to the first coding we only encode the minima or the maxima (one envelope). However, EMD is a numerical method that is prone to numerical errors that may persist in the decomposition as extra IMFs. Thus, IMFs are, in general, not all truly symmetric with respect to the time axis ($y = 0$) but are symmetric about a parallel line $y = \alpha$. Offsets of five IMFs are shown in Table 2. Provided the offset α is encoded, at the decoder the upper (lower) envelope is reconstructed and the lower (upper) envelope is determined by symmetry about line $y = \alpha$. The advantage of this coding is that bit rate is approximately reduced by half.

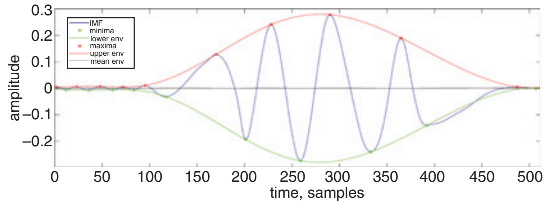

Fig. 3 *Envelopes of an IMF*

Table 2: Offset values of IMFs extracted from ECG

IMF	1	2	3	4	5
α	-0.0001	-0.0026	-0.0008	-0.0011	0.0014

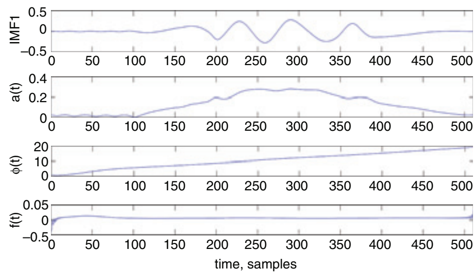

Fig. 4 *IA, IP and IF of an IMF*

IA, IP and IF codings: For some classes of signals such as audio signals [5], both $\mathrm{a}_k(t)$ and instantaneous frequency (IF), $f_k(t) = \dot{\phi}_k(t)/2\pi$ are correlated while IP values are slowly varying. This is well illustrated by Fig. 4. The idea is to encode both IA and IF [4] by linear prediction and to encode IP extrema using scalar quantisation. Remaining IP values can be calculated by linear interpolation. IA and IF can be modelled as follows:

$$a(t) = \sum_{k=1}^{p} c_a(k)a(t-k) + \varepsilon_1(t),\ f(t) = \sum_{i=1}^{L} c_f(i)f(t-i) + \varepsilon_2(t) \qquad (2)$$

where $[c_a(1),\ldots c_a(p)]$ and $[c_f(1),\ldots,c_f(L)]$ are coefficients of the AR model. $\varepsilon_1(t)$ and $\varepsilon_2(t)$ are white noise processes. Thus, we encode $c_a(k)$, $c_f(i)$ and variances of $\varepsilon_1(t)$ and $\varepsilon_2(t)$ using for example Lempel-Ziv encoding. For an IMF, the AR order of IF (IA) is calculated. Since an IMF contains lower frequencies than the previously extracted one (Fig. 2), the order of IF varies from one IMF to the next one. This order can be estimated using the partial autocorrelation coefficient (PAC). Table 3 shows the PAC of the IF of an IMF 5. The order from which the PAC values are constant is identified as the order for IF modelling. The AR order of each IF is given in Table 4.

Table 3: PAC of IF of IMF 5 extracted from ECG signal (Fig. 2)

PAC	0.79	0.55	0.32	0.19	0.063	0.029	0.029	0.029
L	1	2	3	4	5	6	7	8

Table 4: Order of AR model of IF components

IMF	1	2	3	4	5	6	7	8
L	13	9	8	8	6	6	5	5

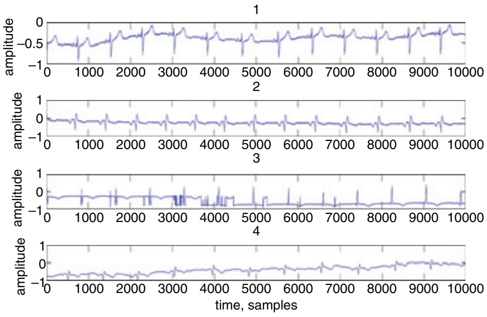

Fig. 5 *Original ECG signals (records 1,2,3,4)*

Table 5: Compression results of ECG signals by extrema, envelope, IA-IP, IA-IF and wavelet codings

	Measure	1	2	3	4
Extrema	CR	31:1	22:1	35:1	39:1
	PRD%	10	**8.3**	4.5	6.3
Envelope	CR	**44:1**	30:1	52:1	**59:1**
	PRD%	10.2	9.4	**3.4**	**6.1**
IA-IP	CR	34:1	26:1	39:1	42:1
	PRD%	9.7	9.8	6.5	7
IA-IF	CR	39:1	**32:1**	**55:1**	51:1
	PRD%	**9.4**	9.7	5.9	6.5
Wavelets (db8)	CR	32:1	20:1	39:1	38.1
	PRD%	10.2	9.9	5.4	6.2

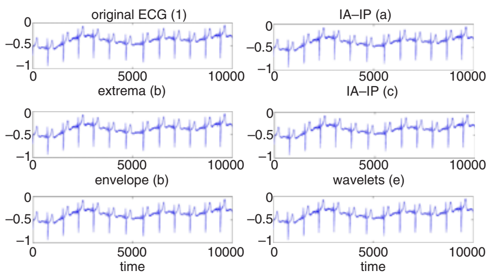

Fig. 6 *Original and reconstructed ECG signal (record 1) by extrema, envelope, IA-IP, IA-IF and wavelets (db8) codings*

Results: We illustrate the potential of EMD codings on four ECG records and compare the results to those of the wavelets (db8) approach. Validation is done through compression ratio (CR) and percentage root mean square difference (PRD). This measure evaluates the distortion between the original and the reconstructed signal. Fig. 5 shows that ECG signals are not linear in their nature, but rather more curvaneous. This is why EMD codings are illustrated on this kind of signals. Table 5 lists achieved PRD and CR where it can be seen that EMD codings achieve better performance in terms of PRD and CR compared to the wavelets approach. The highest CR (59:1) is achieved for record 4 by envelope coding. Also, the best result from the reconstruction error point of view (PRD = 3.4%), obtained for record 3, is achieved by envelope coding. Figs. 6*a*–*d* confirm the results reported in Table 5 where record 1 is well reconstructed with high fidelity with EMD coding methods (Fig. 6*c*). Even though only results of four records are presented, overall envelope coding achieves better performance compared to other methods. This is expected because we encode one out of two envelopes and thus higher CRs are obtained (Fig. 3). However, a large class of ECG signals or other class of signals (e.g. EEG) is necessary to confirm these findings. Even the analysed signals (Fig. 5) have different frequency contents, and the achieved performances show that these signals are well expanded into a reduced number of IMFs, mainly due to the adaptive nature of EMD. No prior assumptions have been made about the number of IMFs of the signals for their expansions and their codings.

Conclusion: In this Letter a new signals coding framework is introduced. Based on IMF properties four strategies are proposed which have shown promising results. Based on EMD these codings are data driven approaches and are computationally simple without pre- or post-processing. The framework is not limited to audio or ECG signals, but can be extended to large classes of signals such as EEG or EMG signals. As future work, we plan to extend the proposed framework to images.

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One or more of the Figures in this Letter are available in colour online.

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